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ELECTROWEAK STRINGS AND FERMIONS

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Z-strings in the Weinberg-Salam model including fermions are unstable for all values of the parameters. The cause of this instability is the fermion vacuum energy in the Z-string background. Z-strings with non-zero fermion densities, however, may still be stable.

1. Electroweak strings

Over the last two decades, extended objects in field theories called solitons have played an important role in particle physics and astrophysics. In certain cases, these solitons possess a conserved topological charge, which guarantees their stability. There are other extended field configurations, however, which are not topological; these "nontopological solitons" are stable for dynamical reasons: they sit at a local minimum of the energy functional. Recently much interest has arisen in the electroweak string,[1,2] a type of nontopological soliton occurring in the Weinberg-Salam model, and in its possible implications for astrophysics and cosmology.[3,4]

The electroweak string is essentially a Nielsen-Olesen cosmic string[5] embedded in the Weinberg-Salam model. Consider a simplified version of the Weinberg-Salam model that includes only bosonic fields:

$$L_{\rm boson} = -\frac{1}{4} W^a_{\mu\nu} W^{a\mu\nu} - \frac{1}{4} F_{\mu\nu} F^{\mu\nu} + \left| D^L_\mu \Phi \right|^2 - \lambda \left(\Phi^\dagger \Phi - \frac{\eta^2}{2} \right)^2 \tag{1}$$

where $W^a_{\mu\nu}$ and $F_{\mu\nu}$ are field strength tensors for the $\mathrm{SU}(2)_L$ and $\mathrm{U}(1)_Y$ gauge fields W^a_μ and B_μ respectively, and $\Phi = \binom{\phi_1}{\phi_0}$ is the complex Higgs doublet. Gauge-covariant derivatives are given by

$$D^L_\mu = \partial_\mu - \frac{ig}{2} \tau^a W^a_\mu - \frac{ig'}{2} Y B_\mu, \qquad D^R_\mu = \partial_\mu - \frac{ig'}{2} Y B_\mu, \tag{2}$$

where Y is the hypercharge of the field on which the derivative acts. The electroweak string, or "Z-string,"[2] is the field configuration

$$\Phi = \frac{\eta f(\rho)}{\sqrt{2}} \mathrm{e}^{i\phi} \begin{pmatrix} 0 \\ 1 \end{pmatrix}, \qquad \begin{pmatrix} Z^1 \\ Z^2 \end{pmatrix} = \frac{2 v(\rho)}{\alpha \rho} \begin{pmatrix} -\sin\phi \\ \cos\phi \end{pmatrix}, \tag{3}$$

all other fields vanishing, where $f(\rho)$ and $v(\rho)$ obey the Nielsen-Olesen equations[5]

$$\begin{aligned} f'' + \frac{f'}{\rho} - (1-v)^2 \frac{f}{\rho^2} + \lambda\eta^2(1-f^2)f &= 0, \\ v'' - \frac{v'}{\rho} + \frac{\alpha^2\eta^2}{4} f^2(1-v) &= 0 \end{aligned} \tag{4}$$

with boundary conditions

$$f(0) = v(0) = 0, \qquad f(\rho) \underset{\rho\to\infty}{\longrightarrow} 1, \qquad v(\rho) \underset{\rho\to\infty}{\longrightarrow} 1. \tag{5}$$

Recall that $Z_\mu = \cos\theta_W \, W_\mu^3 - \sin\theta_W \, B_\mu$ and $\alpha = \sqrt{g^2 + g'^2}$.

The main question is whether the Z-string field configuration is stable. Because the existence of electroweak strings is due to energetic rather than topological reasons, their stability is dependent on the precise values of the parameters in the theory. One possible mode of instability is that the upper component ϕ_1 of the Higgs field may develop a non-zero value, allowing the Z-string to unwind. (There are other modes of instability as well.) To determine whether the string is stable to small perturbations in this direction, one computes the change in the bosonic field energy

$$\Delta E_{\mathrm{boson}}[\phi_1] = E_{\mathrm{boson}}[f, v; \phi_1] - E_{\mathrm{boson}}[f, v; 0]. \tag{6}$$

If the Z-string is stable, $E_{\mathrm{boson}}[f, v; 0]$ is a local minimum of the energy functional, and $\Delta E_{\mathrm{boson}}[\phi_1]$ will be quadratic with a positive coefficient for any perturbation ϕ_1. To determine whether this is so, one inserts the ansatz $\phi_1 = (\eta/\sqrt{2}) g(\rho) \mathrm{e}^{i\omega t}$ into the equations of motion to obtain the eigenvalue equation

$$-g'' - \frac{g'}{\rho} + (\cos 2\theta_W)^2 v^2 \frac{g}{\rho^2} + \lambda\eta^2(f^2 - 1)g = \omega^2 g. \tag{7}$$

If this equation has no negative ω^2 eigenvalues, the Z-string is stable under all perturbations in ϕ_1. Hindmarsh calculated the eigenvalues numerically for the special case $\sin^2\theta_W = 1$ and found that the Z-string is only stable when the Higgs mass is less than the Z-boson mass.[6] A more involved analysis[7] of stability under more general perturbations revealed that in addition the Z-string is stable only for $\sin^2\theta_W$ close to unity, a region that obviously does not include the physical world.

Various authors have tried to increase the range of stability of the Z-string by changing the gauge or Higgs sectors of the model.[3,8,9] Another idea, familiar from the study of nontopological solitons, is to add particles that gain their mass from the Higgs mechanism. These particles remain massless at the center of the string where the Higgs field vanishes, and their presence at the core would resist the string's dissolution, because that would increase their energy. Indeed, the presence of charged scalar bound states was shown to lower the value of $\sin^2\theta_W$ for which the string is stable.[8]

It has been suggested[8] that a similar enhancement of stability could be attained by using fermion bound states on the Z-string. The existence of Z-string zero modes, fermion states localized on the string with zero energy, lends support to this idea.[10,11,12] Another advantage of this suggestion is that fermions are already contained in the standard electroweak model. In the following sections, we consider the effect of standard model fermions on the stability of the Z-string.

2. Fermion zero modes

To discover the effect of fermion fields on the Z-string, we must first determine the fermion spectrum in the background of a Z-string. The Lagrangian of the Weinberg-Salam model, including fermions, is

$$L = L_{\rm boson} + \sum L_{\rm quark} + \sum L_{\rm lepton}. \tag{8}$$

Each quark doublet contributes a term

$$\begin{aligned} L_{\rm quark} &= \overline{\Psi}^L i\not{D}^L \Psi^L + \overline{\psi}_+^R i\not{D}^R \psi_+^R + \overline{\psi}_-^R i\not{D}^R \psi_-^R \\ &\quad - G_+ \left(\overline{\Psi}^L \tilde{\Phi} \psi_+^R + \overline{\psi}_+^R \tilde{\Phi}^\dagger \Psi^L \right) - G_- \left(\overline{\Psi}^L \Phi \psi_-^R + \overline{\psi}_-^R \Phi^\dagger \Psi^L \right) \end{aligned} \tag{9}$$

where $\Psi^L = \binom{\psi_+^L}{\psi_-^L}$, and $\tilde{\Phi} = i\tau^2\Phi^* = \binom{\phi_0^*}{-\phi_1^*}$. Each lepton doublet contributes the same term, absent any pieces containing ψ_+^R. We neglect interfamily mixing.

The Dirac equation in the Z-string background has the form

$$\begin{aligned} \gamma^\mu(i\partial_\mu - \frac{\alpha\ell_\pm}{2} Z_\mu)\psi_\pm^L - m_\pm f(\rho) e^{\mp i\phi} \psi_\pm^R &= 0, \\ \gamma^\mu(i\partial_\mu - \frac{\alpha r_\pm}{2} Z_\mu)\psi_\pm^R - m_\pm f(\rho) e^{\pm i\phi} \psi_\pm^L &= 0, \end{aligned} \tag{10}$$

where $m_\pm = G_\pm \eta/\sqrt{2}$, $\ell_\pm = (y \pm 1)\sin^2\theta_W \mp 1$, and $r_\pm = (y \pm 1)\sin^2\theta_W$, with y the hypercharge of the left-handed doublet Ψ^L. This equation has zero-energy modes,[13,10,11,12] which obey $\gamma_0\gamma_3\psi_\pm = \pm\psi_\pm$. Using the chiral representation for the Dirac matrices

$$\gamma^5 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}, \quad \gamma^0 = \begin{pmatrix} 0 & -1 \\ -1 & 0 \end{pmatrix}, \quad \gamma^i = \begin{pmatrix} 0 & \tau^i \\ -\tau^i & 0 \end{pmatrix}, \tag{11}$$

and recalling that $\psi_\pm^L = \frac{1}{2}(1-\gamma^5)\psi_\pm$ and $\psi_\pm^R = \frac{1}{2}(1+\gamma^5)\psi_\pm$, one may write the zero mode solutions as

$$\psi_{\pm,0} = e^{\pm\int_0^\rho [\ell_\pm v(\rho')/\rho'] d\rho'} \begin{pmatrix} \frac{i}{m_\pm f(\rho)} P'_\pm(\rho)\chi_\pm \\ P_\pm(\rho)\chi_\mp \end{pmatrix} \tag{12}$$

where $\chi_+ = \binom{1}{0}$, $\chi_- = \binom{0}{1}$, with $P_\pm(\rho)$ obeying the equation

$$P''_\pm - \frac{f'}{f}P'_\pm \pm \frac{(\ell_\pm + r_\pm)v}{\rho}P'_\pm - m_\pm^2 f^2 P_\pm = 0 \tag{13}$$

and normalized by

$$\int \mathrm{d}^3x\, \mathrm{e}^{\pm 2\int_0^\rho [\ell_\pm v(\rho')/\rho']\mathrm{d}\rho'} \left[P_\pm^2 + \left(P'_\pm/m_\pm f\right)^2\right] = 1 \tag{14}$$

For the neutrino ($m_+ = 0$, $\ell_+ = -1$), Eq. (13) has the simple solution $P_+ = 1$, but by Eq. (5), $\psi_{+,0} \underset{\rho\to\infty}{\longrightarrow} 1/\rho$, so the zero mode is not normalizable, at least for a straight infinite string (but see ref. 11). Eq. (13) has the explicit solution

$$P_\pm(\rho) = N\mathrm{e}^{-\int_0^\rho m_\pm f(\rho')\mathrm{d}\rho'} \tag{15}$$

for the special case $y = 0$ and $\cos^2\theta_W = \frac{1}{2}$.

The existence of zero modes generates a 2^N-fold degeneracy of the Z-string ground state, where N is the number of quark and charged lepton flavors. The ground state of the string will have the global quantum numbers of each fermion flavor,[14] either $\frac{1}{2}$ or $-\frac{1}{2}$, depending on whether the corresponding zero mode is occupied or not. The occupation of the zero modes will not alter the Nielsen-Olesen equations (4) for the string profile, because the fermion source term for the ϕ_0 and Z_ϕ fields vanishes for the zero modes.

In the $(3+1)$-dimensional context of the Z-string, the zero-energy solution (12) generates a whole family of solutions of the Dirac equation

$$\psi_{\pm,p}(\rho, z, t) \;=\; \mathrm{e}^{ipz - i\epsilon_{\pm,p}t}\;\psi_{\pm,0}(\rho) \tag{16}$$

with energies

$$\epsilon_{\pm,p} = \pm p. \tag{17}$$

These solutions correspond to massless chiral fermions confined to the Z-string; the up-type quarks run up the string (in the $+z$ direction) and the down-type quarks and charged leptons run down the string (in the $-z$ direction) at the speed of light. In addition to these "massless" solutions of the Dirac equation, there are many "massive" solutions, whose energies are separated from zero by a finite gap.

3. Fermion vacuum energy

What effect do the fermion zero modes described in the previous section have on the stability of the Z-string? Earnshaw and Perkins[10] pointed out that the fermion zero mode provides a non-vanishing source term in the equation of motion for ϕ_1. This violates the "Vachaspati existence criterion"[2] and would appear to imply that the Z-string configuration with $\phi_1 = 0$ is not an extremum of the energy. Such a conclusion, however, would be premature.

The reason that the zero modes are a source for ϕ_1 is that the presence of a non-zero value of ϕ_1 lifts the degeneracy between the $\psi_{+,0}$ and $\psi_{-,0}$ zero modes, one linear combination of the zero modes shifting up and the orthogonal combination shifting

down. The lower eigenstate is filled in the ground state of the Z-string, so its descent lowers the Z-string energy. Before drawing any conclusions about the overall stability of the Z-string, however, we must determine the effect of the ϕ_1 perturbation on the rest of the fermion eigenenergies.

The effective energy of the Z-string ground state is

$$E_{\text{effective}} = E_{\text{boson}} + E_{\text{fermion}} \tag{18}$$

where E_{boson} is the bosonic field energy and E_{fermion} the fermion vacuum energy in the Z-string background (*i.e.*, the energy of the filled Dirac sea). The change $\Delta E_{\text{boson}}[\phi_1]$ under a small perturbation ϕ_1 was considered above (6); the change in the fermion vacuum energy is

$$\Delta E_{\text{fermion}}[\phi_1] = \sum_{\epsilon_{+,n}<0} \Delta\epsilon_{+,n}[\phi_1] + \sum_{\epsilon_{-,n}<0} \Delta\epsilon_{-,n}[\phi_1] + \delta E[\phi_1] \tag{19}$$

where $\Delta\epsilon_{\pm,n}[\phi_1]$ denotes the shift of the Z-string Dirac eigenenergies $\epsilon_{\pm,n}$ under the perturbation, and the sum is over negative-energy eigenvalues only.

We compute the eigenvalue shifts $\Delta\epsilon_{\pm,n}[\phi_1]$ perturbatively in ϕ_1. Because the perturbation is off-diagonal in the $+$ and $-$ fields, the leading shift is second order, and the change in fermion vacuum energy is

$$\begin{aligned}\Delta E_{\text{fermion}}[\phi_1] \quad &= \quad \sum_{\epsilon_{+,n}<0}\sum_{\epsilon_{-,m}>0} \frac{\left|\int \mathrm{d}^3x \left(G_-\overline{\psi}^R_{-,m}\psi^L_{+,n} - G_+\overline{\psi}^L_{-,m}\psi^R_{+,n}\right)\phi_1^*\right|^2}{\epsilon_{+,n}-\epsilon_{-,m}} \\ &\quad + \quad \sum_{\epsilon_{-,n}<0}\sum_{\epsilon_{+,m}>0} \frac{\left|\int \mathrm{d}^3x \left(G_-\overline{\psi}^L_{+,m}\psi^R_{-,n} - G_+\overline{\psi}^R_{+,m}\psi^L_{-,n}\right)\phi_1\right|^2}{\epsilon_{-,n}-\epsilon_{+,m}} + \delta E[\phi_1].\end{aligned} \tag{20}$$

The sums over intermediate energy eigenstates $\epsilon_{\pm,m}$ include only positive-energy states; the contributions from negative-energy intermediate states cancel between the two sums. The sums in Eq. (20) diverge in the ultraviolet. The Z-string is not responsible for this, for the same divergence occurs in the usual constant field background. In that case, the divergence is cancelled by adding a counterterm $\delta E[\phi_1]$. The same counterterm will suffice to render $\Delta E_{\text{fermion}}[\phi_1]$ ultraviolet finite.

Let us evaluate the shifts in eigenenergies corresponding to the massless solutions (16) more explicitly. First, restrict the perturbation ϕ_1 to a constant (complex) value $\eta g/\sqrt{2}$ over the region where the zero mode wavefunction (12) is appreciable (but let $\phi_1 \to 0$ as $\rho \to \infty$). Second, noting that E_{fermion} is proportional to the length of the string (as is E_{boson}), consider a Z-string of length L. Periodic boundary conditions on the fermion wavefunctions restrict the z-momenta to $p = 2\pi n/L$. Taking L large, the sum of the energy shifts of the massless states becomes

$$\Delta E'_{\text{fermion}}[g] = -\frac{L}{\pi}|g|^2|A|^2 \int_0^\Lambda \frac{\mathrm{d}p}{2p} \tag{21}$$

with

$$A = 2\pi i L \int \rho \mathrm{d}\rho \frac{(P_+P_-)'}{f} \exp\left(\int_0^\rho \left[(\ell_+ - \ell_-)v(\rho')/\rho'\right] \mathrm{d}\rho'\right) \tag{22}$$

where we have included only massless intermediate energy states. Among this subset of intermediate states, a selection rule ensures that the perturbation only couples the eigenstate $\psi_{\pm,p}$ to the eigenstate $\psi_{\mp,-p}$.

The integral over momenta (21) diverges both in the ultraviolet and in the infrared. As previously noted, the ultraviolet divergence is cancelled by counterterms; the infrared divergence signals the breakdown of the perturbative expansion when the energy denominator $2p$ becomes smaller than the perturbation g. We redo the calculation for states with small p, now treating p as part of the perturbation. The unperturbed states are now degenerate; degenerate perturbation theory yields the perturbed energies

$$\begin{vmatrix} \epsilon - p & gA \\ g^*A^* & \epsilon + p \end{vmatrix} = 0 \qquad \Rightarrow \qquad \epsilon = \pm\sqrt{p^2 + |gA|^2} \tag{23}$$

As mentioned above, the degenerate zero mode ($p = 0$) states are resolved into $\epsilon = \pm|gA|$. This improved calculation yields an infrared-finite result

$$\begin{aligned} \Delta E'_{\mathrm{fermion}}[g] &= -\frac{L}{2\pi} \int_{-\Lambda}^{\Lambda} \mathrm{d}p \left(\sqrt{p^2 + |gA|^2} - |p|\right) \\ &\underset{\Lambda\to\infty}{\longrightarrow} -\frac{L}{4\pi} |gA|^2 \left[1 + \log\left(\frac{4\Lambda^2}{|gA|^2}\right)\right] \end{aligned} \tag{24}$$

The ultraviolet divergence is absorbed by the counterterm, leading to a completely finite expression for the change in the fermion vacuum energy (per unit length) under the perturbation ϕ_1:

$$\Delta E_{\mathrm{fermion}}[g] = L\left(\frac{|A|^2}{4\pi}|g|^2 \log|g|^2 + C_{\mathrm{f}}|g|^2\right) \tag{25}$$

The coefficient C_{f} receives contributions from the shifts of the massive Dirac eigenvalues as well as from the finite part of the counterterm. Each quark doublet contributes a term of the form (25) to the fermion vacuum energy (with different values of A); the charged leptons do not contribute because their eigenenergies are not shifted by the perturbation (in the absence of normalizable neutrino zero modes).

The change in the bosonic field energy for the perturbation we are considering has the form[2]

$$\Delta E_{\mathrm{boson}}[g] = L C_{\mathrm{b}} |g|^2 \tag{26}$$

where the sign of C_{b}, which depends on the parameters of the model, determines whether the bosonic Z-string (without fermions) is stable in the direction of this

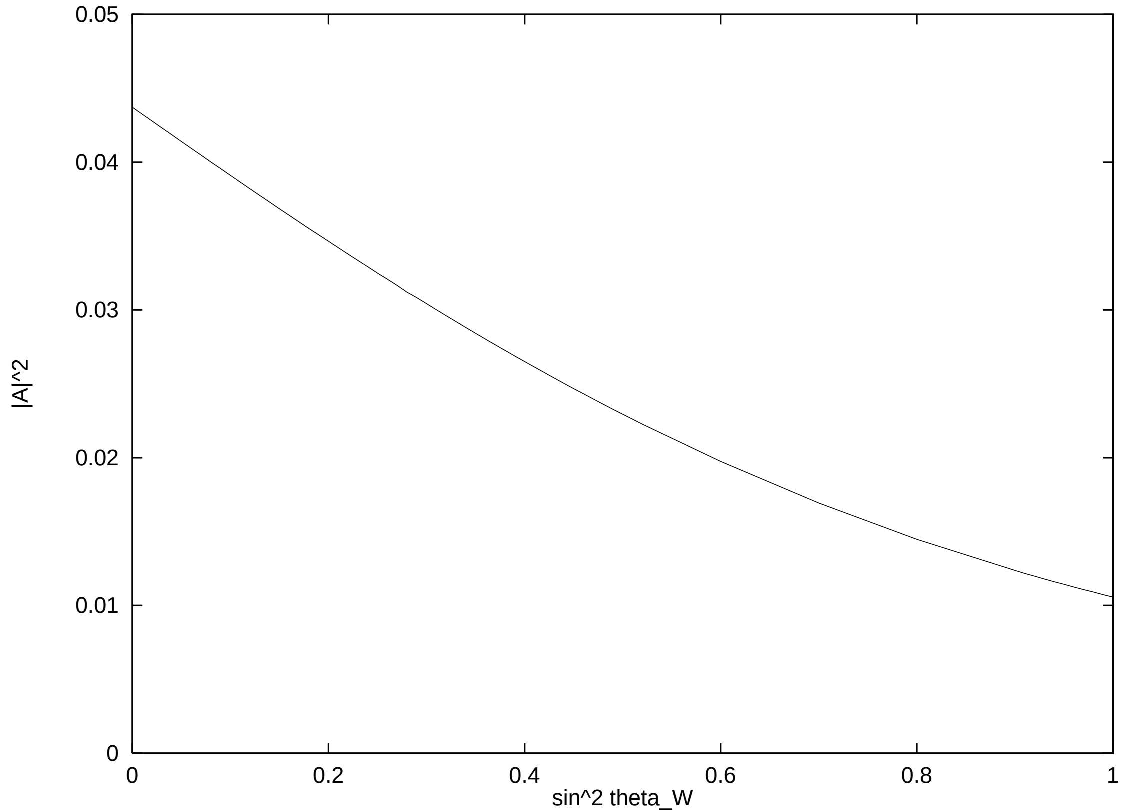

Fig. 1. The value of $|A|^2$ as a function of $\sin^2\theta_W$.

perturbation. Thus, the effective energy of the ground state of the Z-string is

$$E_{\text{effective}}[g] = E_{\text{effective}}[0] + L\left(C|g|^2 + \frac{1}{4\pi}|g|^2\log|g|^2 \sum_{\text{quark}} |A|^2\right) \tag{27}$$

Observe first that $|g| = 0$ is an extremum of this expression, so even in the presence of fermions the Z-string configuration (3) with $|\phi_1| = 0$ remains a solution of the equations of motion. This extremum, however, is necessarily a maximum, regardless of the value of C. (A similar phenomenon occurs in two dimensions.[15]) Hence, the Z-string ground state is unstable to perturbations in ϕ_1 for *all* values of the parameters of the Weinberg-Salam model.

To determine the coefficient of the $|g|^2\log|g|^2$ term in the fermion vacuum energy, we have numerically computed the zero-mode wavefunctions (12) for the various quarks, and used them to calculate A using Eq. (22). The value of $|A|^2$ is a monotonically increasing function of the quark masses, with the third generation of quarks making the dominant contribution to $\sum_{\text{quark}}|A|^2$. In Fig. 1, we have plotted $|A|^2$ for the top-bottom quark pair as a function of $\sin^2\theta_W$, taking $m_{\text{bottom}} = 5$ GeV, $m_{\text{top}} = 175$ GeV, and, arbitrarily, $m_{\text{Higgs}} = m_Z$. (The value of $|A|^2$ depends only

weakly on the Higgs mass.) Thus, for $\sin^2\theta_W = 0.23$, the value of the coefficient $\sum_{\rm quark}|A|^2$, including a color factor of three, is of order 0.1.

For a Z-string of large but finite length L, the fermion energy (24) becomes a sum over momenta, given by

$$\Delta E'_{\rm fermion}[g] = -|gA| - \frac{L|gA|^2}{2\pi}\left[\log\left(\frac{L\Lambda}{2\pi}\right) + \gamma\right] + \cdots \tag{28}$$

(where $\gamma = 0.577\ldots$) in the limit $g \ll 1/L$. The leading term linear in g can be cancelled by populating both (or neither) zero modes. The subleading term then contributes energy per unit length quadratic in g with a negative coefficient that diverges as $\log L$. Thus, regardless of the bosonic contribution (26), the Z-string ground state is unstable for large L, the same conclusion reached above.

4. Conclusions and Outlook

It has been speculated that the presence of fermions would enhance the stability of the Z-string. We have shown[16] that, on the contrary, standard model fermions *destabilize* Z-strings. More precisely, the lowest-energy (or ground) state of the Z-string is always a local maximum of the energy functional with respect to (at least) one of the modes of instability. The ground state of the Z-string is therefore unstable for *all* values of the parameters of the Weinberg-Salam model.

This instability results from the fermion vacuum energy, which also has an important effect on other types of solitons.[17] One cannot consistently consider the effects of positive-energy fermion states without also taking account of the (filled) negative-energy states, particularly because, with the existence of zero modes, there is no gap between them. We have shown that the contribution to the energy functional of the filled Dirac sea, *i.e.*, the fermion vacuum energy, is a local maximum for the Z-string.

This does not necessarily mean that there exists no stable nontopological string configuration. A (locally) stable string with ϕ_1 slightly displaced from zero may exist, though that remains to be demonstrated. What we are saying is that the simple Nielsen-Olesen string embedded into the Weinberg-Salam model with all other fields vanishing is necessarily unstable.

Most attempts to increase the stability of the Z-string do so by increasing the coefficient C in Eq. (27); this will not work here since no coefficient, however positive, can outweigh the negative curvature at $|g| = 0$ caused by the $|g|^2\log|g|$ term. The only way to overcome this term is to occupy some of the *positive*-energy fermion states. This requires not just a single fermion (as in the case of nontopological solitons) but rather a finite density of fermions along the Z-string. If the string holds ζ positive-energy fermions per unit length of types $+$ and $-$, the effective energy will change by

$$\Delta E[g] \quad = \quad \frac{L}{2\pi}\int_{-2\pi\zeta}^{2\pi\zeta} {\rm d}p\left(\sqrt{p^2+|gA|^2} - |p|\right)$$

$$\xrightarrow[|gA|\ll 2\pi\zeta]{} \quad \frac{L}{4\pi}|gA|^2\left[1+\log\left(\frac{16\pi^2\zeta^2}{|gA|^2}\right)\right] \tag{29}$$

If the Z-string carries a non-zero density of quarks of *each* flavor, the change in energy will cancel the $|g|^2 \log |g|^2$ piece in Eq. (27), rendering the total energy proportional to $|g|^2$. It is therefore possible that a higher-energy state of the Z-string, with some finite quark density, could be stable.

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